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An exploration of university physics students' epistemological mindsets towards the understanding of physics equations

Abstract

Students' attitudes and beliefs about learning have been shown to affect learning outcomes. This study explores how university physics students think about what it means to understand physics equations. The data comes from semi-structured interviews with students from three Swedish universities. The analysis follows a data-based, inductive approach to characterise students' descriptions of what it means to understand equations in terms of epistemological mindsets (perceived critical attributes of a learning, application, or problem-solving situation that are grounded in epistemology). The results are given in terms of different components of students' epistemological mindsets. Relations between individuals and sets of components as well as differences across various stages of students' academic career are then explored. Pedagogical implications of the findings are discussed and tentative suggestions for university physics teaching are made.

INTRODUCTION

In the higher education student learning community, particularly in physics education research, there is a growing interest in what phenomenologists call the *natural attitude* – the habitual world that we live in. In terms of enhancing the understanding of student learning this interest is underpinned by epistemology (see review by Hofer & Pintrich, 1997). For example, epistemologically associated notions of *framing* and *resources* (e.g. Redish, 2003) and *relevance structure* (e.g. Marton & Booth, 1997) are used to portray what is perceived to be *necessary, relevant and important for a*

given situation. In this article we argue for characterising the epistemological essence of this idea as a *mindset*. Epistemologically based research such as that done by Hammer (1994), Roth and Roychoudhury (1994), Redish, Saul and Steinberg (1998), May and Etkina (2002), Adams et al. (2006), and Lising and Elby (2004) has indicated that physics students' learning is significantly related to their perceptions about the nature of physics and about physics learning and knowledge.

Much of university physics teaching and learning is centred around an extensive use of what are colloquially known as *physics equations*. A small number of studies have explored students' understanding of the symbolic structure of equations. For example, Kieran (1981) has investigated students' interpretations of the equal sign, concluding that many students view the equal sign as meaning "do something", although Kieran stresses that it is not clear whether this interpretation is harmful. Govender's (1999) study explored students' ways of interpreting sign conventions used in introductory classical mechanics equations. The main finding of this study was that students found it difficult to interpret the positive or negative sign allocated to the vector components. Hence, students found it difficult to transfer from one-dimensional motion to two- and three-dimensional motion. Other studies have investigated students' understanding of the variables in equations. Clement, Lochhead and Monk (1981) found that students had difficulties in translating from a verbal representation to a mathematical representation in terms of algebraic symbols. Rozier and Viennot (1991) identified that students found it hard to parse the relationship between variables in multivariable problems, and Steinberg, Wittmann and Redish (1997) found student difficulties in relation to equations involving functions of more than one variable. Sherin (2001) has looked at students' understanding of equations in terms of how students construct equations using basic templates. Sherin argues that, when presented with a physics situation and asked to come up with equations that describe this situation, students make use of various structural templates for equations in order to find an equation that describes the situation appropriately.

So while a range of work has looked at a variety of students' experiences of equations, not much of this has had a focus on equations from an epistemological perspective. One notable exception, Redish, Saul and Steinberg (1998), has looked at students' expectations of equations as part of a more general study of students' expectations of physics. In their study, they conclude that many students carry expectations that the mathematical aspects of an equation are important and that many introductory physics students "fail to see the deeper physical relationships present in an equation and instead use the math in a pure arithmetic sense – as a way to calculate numbers" (Redish, Saul & Steinberg, 1998, p. 11).

The work reported in this article draws on the research described earlier and adds to the understanding of students' experience of equations from an epistemological perspective by starting with an exploration of students' epistemological mindsets towards what it means to understand physics equations. This is done in a case study involving 20 physics students from three Swedish universities across various levels in their education; ranging from first-year undergraduate to the PhD level.

Mindsets

Following the seminal constructivist work of Piaget with children (e.g. 1950) that showed how epistemology was related to learning, a strong interest in *what students are aware of in the learning situation* started to emerge in the physics education research community. Arguably the most significant boost to this interest has been Perry's (1970) and Baxter Magolda's (1992) work on epistemological influences on the learning of college students, the comprehensive modelling of an *anatomy of awareness* (Marton & Booth, 1997) and the debate around the complimentary ideas of *phenomenological primitives* and *ways of experiencing* (diSessa, 1993a; 1993b; Marton, 1993). Then distinctive work situated around, for example, how teachers may be creating further hurdles in physics learning (Linder, 1992) and how students' learning of physics may be affected

by epistemology (Hammer, 1995; Hammer & Elby, 2000) has contributed to considerable growth in interest in epistemological issues (for example, Redish, 2003).

An ongoing debate exists over how to model a person's epistemology to better understand and inform student learning. In general, the most common models are beliefs, traits/styles, and resources (Elby & Hammer, 2001; Hofer & Pintrich, 1997). Resources were introduced by Elby and Hammer (2001) in an attempt to achieve a more fine-grained description of epistemological beliefs in the same way as diSessa (1993a) introduced p-prims in order to obtain a more fine-grained description of conceptions. Resources are context-dependent building blocks of epistemological reasoning and do not exist in isolation but are related and coordinated in what is known as coordination classes (diSessa & Sherin, 1998). Apart from the magnification scale, these models also differ as to the form of the epistemology, whether it is explicit or implicit for the student, and how context-dependent it is. One additional model makes no claim as to form or more fine-grained structure and this is the phenomenographic *relevance structure of the learning situation* model. Marton and Booth (1997, p.143) describe relevance structure as being the "person's experience of what the situation calls for, what it demands. It is a sense of aim of direction, in relation to which different aspects of the situation appear more or less relevant". At the same time the phenomenographic perspective is firmly anchored in its associated empirical findings that reveal variation in ways of experiencing as being related to context.

In our research, we were interested in exploring students' epistemological views of what it means to understand physics equations. In the first stage of this research we were simply interested in describing the range of epistemological views that could be found among the students. This meant that the notion of resources and coordination classes involved a too detailed description with too much internal structure in relation to what we were looking for. Thus for our research question we initially found it to be most fruitful to draw on the notion of *relevance structure*. However, when analyzing data, we did find a more fine-grained structure in students' epistemological views, although without the internal links which characterize resources in coordinate classes. Furthermore, as far as relevance structure is concerned, it is a rather general concept which involves *no* fine-grained structure and it does not explicitly refer to epistemology. Therefore, we decided that we needed something different from resources and relevance structure to characterize students' descriptions. For this, we came up with the notion of a mindset which we define as *perceived critical attributes of a learning, application, or problem-solving situation that are grounded in epistemology*.

As an example, a student who believes it to be important to link equations to everyday life situations as well as to know when to apply an equation is likely to explicitly focus on these aspects when presented with physics equations. Thus, establishing links to everyday life as well as determining when to appropriately use the equation would be an integral part of the mindset of such a student. However, for another student who believes it to be important to link equations to everyday situations, being able to know when to use the equation is not necessarily part of the students' mindset. Thus, there are no obvious links between different components of students' mindsets. Our study does not explicitly attempt to explore the relationships or links between components of mindsets, but characterizes the different aspects of what it means to understand an equation in terms of mindsets. Examples to help further clarify the notion of a mindset are provided by the interview excerpts in the results section.

RESEARCH QUESTIONS

In order to begin an investigation of students' epistemological mindsets towards the understanding of equations, we focus on two main research questions in this study:

- When students say that they understand an equation, how do they describe what that means to them, and how can these descriptions be characterised in terms of epistemological mindsets?
- Are similar epistemological mindsets observable for students at various stages in their academic career?

METHOD

In order to explore the research questions presented in the previous section in as fruitful a way as possible, we decided to carry out an exploratory case study of students' epistemological mindsets towards what it means to understand an equation. Twenty voluntary physics students from three different Swedish universities were interviewed using a mixed mode semi-structured interviewing strategy involving either face-to-face or e-mail interviews (c.f. Meho, 2006). Seven of the interviewees were first year undergraduate students, nine were second or third year undergraduate students and four were PhD students.

As described by Merriam (1988) a case study is "a detailed examination of one setting, or a single subject, a single depository of documents, or one particular event". In our case, we examined epistemological mindsets towards the understanding of equations in a group of twenty students. We decided to frame this study as a case study and to use interviews because we wanted to explore in detail the experiences of each individual student participating in the study. This would enable us to map, characterise and further analyse the students' epistemological mindsets.

Each face-to-face interview lasted approximately twenty minutes and began with some introductory discussion centred around the nature of physics and the role of mathematics in physics. The purpose of this introductory discussion was to set the context to physics and physics equations. After this discussion, the main question that we asked the students and were interested in exploring was: "When you say or feel that you understand an equation, what does that mean?". Associated follow-up questions such as "What do you mean by that?" and "Are there other things you believe are important?" were used for further probing, clarification, and to allow students to elaborate on their answers. A similar approach was used for the e-mail interviews.

The overall aim of the interviews was to engage students in an in-depth discussion to explore their view of what it means to understand physics equations. It is important to stress that we were not trying to capture students' first spontaneous responses, but to take all aspects of what the students viewed as important for understanding physics equations into account. All of the face-to-face interviews were audio-recorded and selected parts were transcribed verbatim. These transcripts, together with the complete discussions from the e-mail interviews, formed the data for this study. Since several of the students are Swedish, some of the examples from the data presented in this study have been translated from Swedish to English and where this is the case this is clearly marked after the excerpt.

To add an additional dimension to the study we included students at various levels in their education, ranging from first-year undergraduate students to PhD students. By having students from various educational stages, a cross-sectional case study was created, where we could both characterise students' epistemological mindsets towards the understanding of equations and analyse whether there is a difference in the mindsets for students at different stages in their academic career.

The principal aim of the data analysis process was to characterise students' descriptions of what it means to understand a physics equation when they feel that they have understood it and then to take a closer look at these results. For this purpose we used what could be described as a standard, qualitative, data-based inductive analysis. As described by Bogdan and Biklen (1982, p.145)

this involves “working with data, organizing it, breaking it into manageable units, synthesizing it, searching for patterns, discovering what is important and what is to be learned, and deciding what you will tell others”. The process is in general carried out inductively, i.e., patterns and themes originate from the pool of data.

The first phase of the analysis process involved identifying overall themes in the raw data – a process characterised as “open coding” (cf. Strauss & Corbin, 1990) – and tentatively grouping pieces of data into descriptive categories corresponding to different characterisations of students' descriptions of what it means to understand an equation. The categories were given a descriptive heading and each piece of data was coded with an identification tag involving the origin of the data and to which of the tentative categories it was assigned.

In the next phase the characterisations were iteratively compared, modifying, replacing, splitting or merging the categories until saturation occurred. During this phase there was also a continual cross reference to the full transcripts and the two described steps in the data analysis process were essentially carried out simultaneously in iterative cycles. This process continued until the characterisations stabilised into a comprehensive set of outcomes that well captured the content and richness found in the data.

RESULTS

Using epistemological mindsets as our analytical base, the characterisations we identified during the data analysis process could be best described as generic *components* of epistemological mindsets, where an individual student's epistemological mindset towards the understanding of a physics equation could consist of one or more of these components.

Next, we present and describe these epistemological components, which are later illustrated by examples from the interview data. Then, we discuss in detail which components we could identify in the individual students' epistemological mindsets.

Epistemological components of students' mindsets towards the understanding of physics equations

As far as the characterisations of the components of the students' epistemological mindsets are concerned, we would like to point out a few things. Firstly, the individual students involved in this study are generally in a one-to-many correspondence with the characterisations, reflecting the fact that the epistemological mindset of an individual student involves ingredients from several of the characterisations. Secondly, the order of the components presented should not be interpreted as hierarchical. Some of the epistemological components could be viewed as corresponding to a more fruitful epistemological mindset, but we prefer to view the components as corresponding to complementary parts of an appropriate disciplinary epistemological mindset towards the understanding of an equation. Thirdly, throughout the description of these epistemological components, we use the equation providing the speed of a longitudinal wave in a fluid, $v = \sqrt{B/\rho}$. The use of this equation is only for illustrative and clarification purposes and does not reflect or represent actual excerpts from the interview data.

Epistemological component A – understanding involves being able to recognise the symbols in the equation in terms of the corresponding physics quantities

Here understanding an equation involves being able to recognise what all the symbols in the equation represent in terms of corresponding physics quantities. In the case of $v = \sqrt{B/\rho}$, this would correspond to identifying v as the speed of the wave, B as the bulk modulus and ρ as the density of the fluid through which the wave propagates.

Epistemological component B – understanding an equation involves being able to recognise the underlying physics of the equation

This component involves one or several subcomponents such as knowing what the quantities in the equation mean from a disciplinary physics point of view, what the underlying concepts and principles of the equation are, or being able to know the origin of the equation in terms of how it is derived. If we once again look at the equation $v = \sqrt{B/\rho}$, this could correspond to an understanding of what the speed of a wave means, what the bulk modulus of a substance represents and what density is. It could also involve knowledge of waves (longitudinal waves in particular) as well as an idea of how the equation is and can be derived from more fundamental concepts.

Epistemological component C – understanding involves recognising the structure of the equation

This epistemological component involves understanding how the different quantities in the equation are related to each other and the equation as a whole in terms of where the quantities are situated in the equation and what this infers. Using $v = \sqrt{B/\rho}$ to clarify, this epistemological component would involve considerations of whether it makes sense to have the bulk modulus B in the numerator and the density ρ in the denominator, i.e., does it make sense that the speed of the wave increases if we have a larger bulk modulus B and that the speed increases if we decrease the density? What happens to the speed if we have a bulk modulus that is four times larger?

Epistemological component D – understanding involves establishing a link between the equation and everyday life

Two main types of links to everyday life could be identified in the data. The first type involves situating the equation in an everyday context, by identifying examples and situations in everyday life where the equation applies. The second type of link consists of finding analogies from everyday life that help in appreciating the meaning of the equation. For the first type of link an example would be realising that $v = \sqrt{B/\rho}$ could describe the propagation speed of sound in air. For the second type of everyday linking through analogies one could compare wading through water to walking through air in order to appreciate the dependence of the speed on the density.

Epistemological component E – understanding involves knowing how to use the equation to solve physics problems

Here understanding involves being able to know how to use the equation, i.e., solving physics problems by using the mathematical manipulations that are needed to extract the sought information from the equation. This component also involves identifying which information is sought as well as what other information is available or needed. Once again using $v = \sqrt{B/\rho}$ to clarify, this component would involve being able to use this equation to calculate the speed of longitudinal waves for a fluid with a given bulk modulus and density, or more generally being able to calculate any of the three quantities from the equation given the other two.

Epistemological component F – understanding involves being able to know when to use the equation

This component involves knowledge of the range of validity of the equation, inherent approximations and idealisations and in some cases also what branch of physics the equation is supposed to describe. In the case of $v = \sqrt{B/\rho}$, this would involve knowing for what kind of waves this equation can be used, and for what kind of waves it cannot be used. It would also involve acknowledging factors such as the fact that this equation presumes small amplitudes and linear waves and that the fluid is considered to be a continuum.

Examples illustrating the epistemological components

Before we present examples from the interview data, it is important to once again stress that multiple components can be and generally are part of an individual student's epistemological mindset towards the understanding of an equation. This means that many of the excerpts in this section will therefore illustrate several of the epistemological components. In the next section we provide a detailed description of which components the individual students participating in this study appeared to hold.

If we begin the illustrations with component A, where understanding involves recognising the symbols in the equation in terms of corresponding physics quantities, an illustration of this component is provided by the bold part of the excerpt below. Throughout this section the interviewer is labelled "I" and all the interviewees are labelled "Student".

I: When you feel that you have understood a formula, what does that mean to you?
Student: That I understand a formula... **well, that is when I know what all constants are, what all the symbols mean so to speak... like that is the speed of light and that is the frequency and so...** and what you can get out of it. Isn't most formulae constructed with the aim that one should get... get a particular value... what value does one want and how can one get it? [Original in English]

The same excerpt also illustrates component E (knowing how to use the equation). This can be seen from the last part of the excerpt and in particular from the final sentence with the student wanting to find out what to extract from the equation and how this can be done. Returning to component A (recognising the symbols) yet another illustration is provided by the example below in particular in the bold section.

I: When you say that you understand an equation, what does that mean?
Student: It is when I can see... alright... this part of the formula is... potential energy ... and you understand, OK, what's changing when you're changing the distance and so on. For example, what is x in this formula, what is t in this formula? You have a knowledge of each and every one of the letters... like what's m? Is it the electron mass or some other mass? You have to know which parts are constants and which are variables. **So I understand a formula when I can change all the letters into physical terms.** Then I can start calculating and find an answer. [Original in English]

In this example, we can also find instances of component C (recognising the structure of the equation) – “what's changing when you're changing the distance” – as well as component B (recognising the underlying physics) – “Is it the electron mass or some other mass?”

If we compare component A (recognising the symbols), which we illustrated with typical excerpts above, with component B (recognising the underlying physics) the main difference is that for component B, understanding a physics equation involves a recognition of the underlying physics. This can involve knowing what the physics quantities mean (e.g., what is a wave function), what underlying concepts and principles exist or the way in which the equation can be derived. A nice example, in the case of recognising what the physics quantities mean, is provided by the bold section in the excerpt below.

Student: I understand a formula when I can... when I understand what all of the... all of the things in the formula is. I mean... for example **knowing that this psi is the wave function and what a wave function is.** Of course it is also important to know how to use it and when it should be used, and it is also nice to get an... get an idea of why it is useful. I mean... what is it describing? [Original in Swedish]

In this excerpt, it is also possible to find instances of component A (recognising the symbols in the equation), E (knowing how to use the equation), F (knowing when to use the equation) and perhaps even D (linking to everyday life), although it is not clear whether “what is it describing?” necessarily has to be related to everyday life.

A further illustration of component B (recognising the underlying physics) in the case of students wanting to understand the origin of an equation and how it can be derived is bold below:

Student: For me understanding a formula means that you understand what the formula is all about, what the different variables and constants are and how you can solve for any of these using the formula. As an example the equation for a straight line, $y = kx + m$, that one here knows that the k-value is calculated by dy/dx and how to extract such a value... that one also understands what m means and that one gets the y-value by knowing the rest. Also, that one can find out the m-value if you know the rest and so on... **One should really also be able to derive the formula to understand the background of how it came about and its history.** But if you know the first stuff I think you are pretty far along the way... [Original in Swedish]

Apart from component B, the epistemological mindset involved in the student excerpt above can also be seen to contain component A (recognising the symbols) and component E (knowing how to use the equation).

For component C there is a focus on the *structure* of an equation and the relations between the quantities inherent in the equation in terms of where the quantities are situated and what could be inferred from this. The excerpt below provides a neat illustration of this component.

Student: I believe that I understand a formula when all the terms in the formula feel logical, like increased mass gives larger gravitational force, which means that the mass is in the numerator... increased distance reduces the force so the distance is in the denominator. [Original in Swedish]

Another illustration of component C (recognising the structure) is provided in the excerpt below, which involves the same interest in being able to see how the terms of the equation relate to each other and affect the output from the equation. This particular student also goes beyond this by considering how the context affects the parameters and their relations in the equation, i.e., there is also a presence of component B (recognising the underlying physics).

Student: I think that the most fun is when you can know how things relate to each other, so you don't have any numbers at all, you just have... what's the difference if you change this... turning around the formula. That's a very interesting problem, because then you have to understand the formula very much to know what happens if I change this and... what's the difference if it's a different material... what should I change when I change materials? Is it x or is it t or is it h or what is it? so... yeah, that's when I understand a formula. [Original in English]

In component D, students want to establish a link between the equation and everyday life – either in terms of finding an everyday situation or application for the equation or in terms of using everyday life analogies to help create meaning from the equation. An example is provided by the excerpt below, where the student wants to “put it into reality” and then exemplifies this with an everyday example.

I: What does it mean when you say that you understand a physics formula?

Student: It is when I can take that formula and put it into reality. [...] For example, a

lamp that lights up a wall... when I can use a formula to calculate how much light that hits the wall and how it changes if I move the lamp. [Original in Swedish]

A further example of component D (linking to everyday life) where a student establishes a link in terms of analogies in order to make it easier to conceptualise the equation is provided below.

- I: What does it mean to understand a formula? When you feel that you have understood a formula, what does that mean to you?
- Student: Well... then I'd better use a simple example – Ohm's law. U equals R times I , where U is the voltage, R the resistance and I the current in a conductor. However simple this formula may be, it is not obviously intuitive in electrical contexts, since electrical quantities cannot be perceived by our senses. [Original in Swedish]

This student then goes on to discuss Ohm's law in terms of a water hose – how changing the diameter of the hose can affect the water flow. Here the student is using an everyday analogy (even though the analogy might not be entirely appropriate from a physics point of view) in order to conceptualise or “see” what the equation represents.

In Component E being able to use the equation to do calculations and solve problems is seen to be an important part of the understanding of an equation. An illuminating illustration where understanding involves applying the equation is provided by the excerpt below, where the student views equations as “tools” for solving problems.

- Student: I feel that I understand a formula when I am able to use it to solve problems. They are like tools that you use when you solve problems. [Original in English]

The excerpt below illustrates another student for whom use is an important component of the epistemological mindset towards the understanding of an equation. This student even compares the relevance of the usage of an equation to knowledge of the origin of the equation, considering the former to be a more important component of the understanding of an equation.

- I: What does it mean when you say that you understand a physics formula?
- Student: I guess you can calculate things, and know where the things came from and what it is basically. But... the more important thing is how to use it.
- I: How to make a calculation?
- Student: Yeah, and what problem I'm going to solve with the equation. It is not so important to know who came up with it and how he did when he derived it. [Original in English]

Component E, which we illustrated above, involves focusing on being able to use the equation to solve physics problems. This is different from component F, where students explicitly describe knowledge of *when* the equation can be applied to be important for the understanding of physics equations. The bold text in the excerpt below provides an illustration of component F, where an acknowledgement of *when* the equation can be applied and what range of validity the equation has are considered to be important.

- Student: I believe that I understand a formula when all the terms in the formula feel logical, like increased mass gives larger gravitational force, which means that the mass is in the numerator... increased distance reduces the force so the distance is in the denominator. I should also know what all the variables represent of course and **know when the formula can be applied. Like... Newton's laws of motion one can apply when the velocities are much smaller than the velocity of light.** [Original in Swedish]

Once again several of the components are illustrated with a single excerpt. Apart from component F (knowing when to use the equation) there are also instances of component A (recognising the symbols) and C (recognising the structure) in the epistemological mindset of this student.

A closer look at the epistemological components in terms of individual students

Here we map the epistemological mindset of individual students in terms of which of the epistemological components we could identify as being described by the students to be an important part of understanding an equation.

After a process of identifying components for the individual students, we arrived at Table 1 below, which shows which epistemological components (labelled horizontally from A to F referring to the different epistemological components presented previously) are present for the individual students in this study (labelled vertically from 1 to 20).

Table 1. A mapping of the components identified in the individual students' mindsets. Students 1-7 are first year undergraduate students, students 8-16 are second or third year undergraduate students and students 17-20 are PhD students. The last row involves a counting of the number of occurrences of the components corresponding to each column.

	A Recognise symbols	B Underlying physics	C Structure	D Everyday life	E How to use	F When to use
1	*			*	*	
2	*		*			
3	*				*	
4					*	
5		*			*	
6				*		
7	*	*			*	
8	*		*			*
9				*		
10				*		
11				*		
12	*				*	
13	*	*		*	*	*
14	*					
15					*	*
16				*		*
17	*	*			*	
18		*			*	*
19		*			*	
20		*				
Σ	9	7	2	7	11	5

DISCUSSION

As described earlier, previous research has explored a variety of students' experiences of equations. However, students' understandings of equations have not been extensively explored using an epistemological perspective. Our primary research aim in this study was to explore how students describe what it means to understand an equation and to characterise these descriptions in

terms of epistemological mindsets. We found that students' epistemological mindsets could be seen as being composed of one or more components, reflecting a variation in what is seen as important for the understanding of physics equations. It is encouraging to conclude that these components include many of the aspects that one would hope to find in a disciplinary understanding of a physics equation: knowing what physics quantities are involved, understanding the underlying physics and appreciating the origin, establishing a link to everyday life and knowing how and when to use the equation.

We believe that all of the components identified in this study are integral parts needed to have an appropriate understanding of physics equations. In the study by Sherin (2001), outlined earlier, various structural templates that students use when they attempt to construct physics equations for given situations were described and identified. Ideally, a student would be familiar with and be able to implement all of these templates in order to be able to use equations efficiently to describe various physics situations. In the same vein, ideally the epistemological mindsets of *individual students* towards what it means to understand an equation would involve all of these components in order for the student to have an appropriate understanding of physics equations. However, if we turn to the mapping of the components we identified in the epistemological mindsets of individual students, we can see that this may, all too often, unfortunately not be the case. For example, the epistemological mindset of one of the students involves five of the components (student 13 in table 1), while the mindsets of the rest of the students in this study involve from one to three of the components.

Taking a closer look at the different components a number of interesting points emerge. First, it can be seen from Table 1 that there are no obvious patterns as far as the various components are concerned. There are no groups or pairs of components that exhibit any apparent collective correlations in the sense that no specific groupings emerge across the students. Secondly, it can be seen from the final row in Table 1 that the most frequently occurring component is component E (knowing how to use the equation) which was found in the mindsets of eleven of the students in this study, and component A (recognising the symbols) which was identified for nine of the students. One of the aspects that can be inferred from this – besides that knowing what the symbols represent and how to use an equation are the most common components across the mindsets of the students – is that *how to use* an equation is seen as more important than knowing *when to use* the equation. Only three of the students express a mindset where *how to use an equation* is linked together with *when to use the equation*. It could be argued that when to use an equation is implicit in how to use it, but we could not find any indications of such a relationship in our interview data. The students' central focus on how to use an equation, which we found in our study, corresponds with the findings reported by Redish, Saul and Steinberg (1998), where many students simply view equations as a way to solve problems and to calculate numbers.

A second aim of our study was to explore whether similar mindsets are observable for students at various stages in their academic career. If we once again look at Table 1, but this time compare the components identified for students at different stages in their academic career, it is difficult to discern any obvious differences. Apart from possibly a higher frequency of component B (recognising the underlying physics) and a lower frequency of component D (linking to everyday life) for the PhD students the patterns are similar across all of the students. This could mean that there is no gradual change in the epistemological mindsets towards the understanding of an equation, but due to the small number of students involved in this study these results should at most be viewed as tentative. It is, however, interesting to compare these results to what Redish, Saul and Steinberg (1998) found. In their study they concluded that no improvements in students' expectations of the role of equations could be found after an introductory physics course. In fact, some classes show "a significant and substantial deterioration" (p.11). It would be interesting and pedagogically important to explore further how students' epistemological views evolve, by following a group of

students over a longer period of time, to explore whether students' epistemological mindsets exhibit any transformations as students move through the various stages of their learning of physics.

SUGGESTIONS AND IMPLICATIONS

One possible way of interpreting the results described and discussed in this paper is as a reflection of a learning situation where the students are never or rarely provided with the opportunity or incitement to give epistemological questions such as "What does it mean to understand an equation?" any serious thought. In most cases of traditional physics teaching, it may not be necessary or even beneficial to have an epistemological mindset towards the understanding of equations that involves all of the components identified in this study. Indeed, the results of this study indicate that students' epistemological mindsets generally only involve a small number of components.

The most frequently occurring components found in this study were how to use an equation to solve problems and recognising what the symbols in an equation represent. It is tempting to interpret these results as being a consequence of the traditional way many physics departments still to a large extent view, present and assess student knowledge in physics. To be a successful physics student it is often enough to be able to identify the physics quantities in the equation and know how to use the equation to solve physics problems. This is an unsatisfactory situation from a disciplinary physics point of view, since it turns physics equations into little but mathematical tools – referred to as "dead leaves" by Redish (1994).

We would like to propose holding an in-class discussion of epistemological issues (such as what it means to understand a physics equation) at an early stage of students' learning of physics. Such a discussion could prove fruitful in widening students' epistemological mindsets into mindsets that go beyond just one or a few of the epistemological components. Ideally this discussion should not be implemented as some kind of "patch" in isolation. A richer view of what it means to understand an equation needs to transcend all of the teaching and learning of physics. Apart from initiating a discussion of what it means to understand an equation we also believe it to be necessary and fruitful to present students with situations, tasks and problems that challenge their epistemological mindsets making the need for a wider epistemological view apparent. Unless we actively try to do this it is unlikely that the majority of students will develop the epistemological view that we would like them to have. This claim is supported by the results of this study if we compare the epistemological mindsets of students at different stages in their academic career. As can be seen from Table 1 there may be some indication that students at a late stage of their academic career find an understanding of the underlying physics to be more important than students at an earlier stage of their academic career. There is however no indication of a richer epistemological mindset in terms of a larger number of epistemological components being involved.

It would be interesting to implement the suggestions we presented above in a physics course, to see whether they influence students' epistemology and in particular their mindsets towards the understanding of equations. However, changing students' epistemology is likely to be a difficult process. As an example, May and Etkina (2002) report that "even if the course is structured in an epistemologically favourable way and students do not receive new concepts from authority, some of them still think that they learn from authority" (p.1256). It seems that changing students' epistemologies is easier said than done. That said, we believe it to be necessary to continue to generate and test potentially fruitful ways of establishing an epistemological awareness and an appropriate epistemological view of physics and the learning of physics.

Looking at the implications of this study for educational research, it is important to start by pointing out that the results found in this case study can only be claimed to apply to the particular group of students involved. However, due to the generic, traditional nature of much of the physics

teaching worldwide, a hypothesis would be that to a large extent the same patterns could be found for other groups of physics students in different locations. It would thus be interesting to use similar methods to those in this study to explore whether such a hypothesis holds or whether there are vastly different results for different student populations.

It would also be interesting to further investigate the indications found in this study which suggest that students' epistemological mindsets towards the understanding of equations exhibit no clearly discernable differences for students at various stages in their academic career. It would be exciting to investigate this further for a larger student population or perhaps even better, as suggested earlier, to conduct a longitudinal case study on a student population.

From earlier epistemological research, described in the introductory section, there seems to exist clear relations between epistemological views and student learning. It would be interesting to conduct a similar study in relation to students' understanding of equations, where correlations between students' epistemological mindsets and their approaches when dealing with equations are explored.

This study can also be viewed as a first explorative step in a larger investigation of students' epistemological mindsets of the understanding of physics equations. We hope to use the results of this study in future research to inform a research-based construction of a survey to investigate students' epistemological mindsets of what it means to understand an equation for a large student population.

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